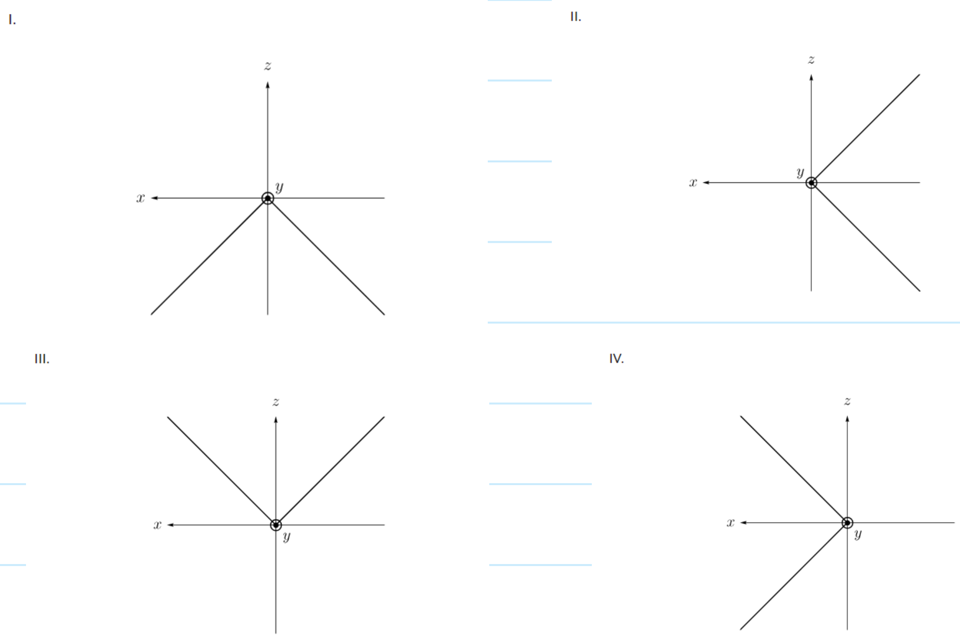
多變數函數的圖形

2021年5月29日　　下午 06:09

Let's start with a simple scenario where the function depends explicitly only on x. When z doesn't explicitly depend on y, we can use a 2D plot to find the full 3D graph.

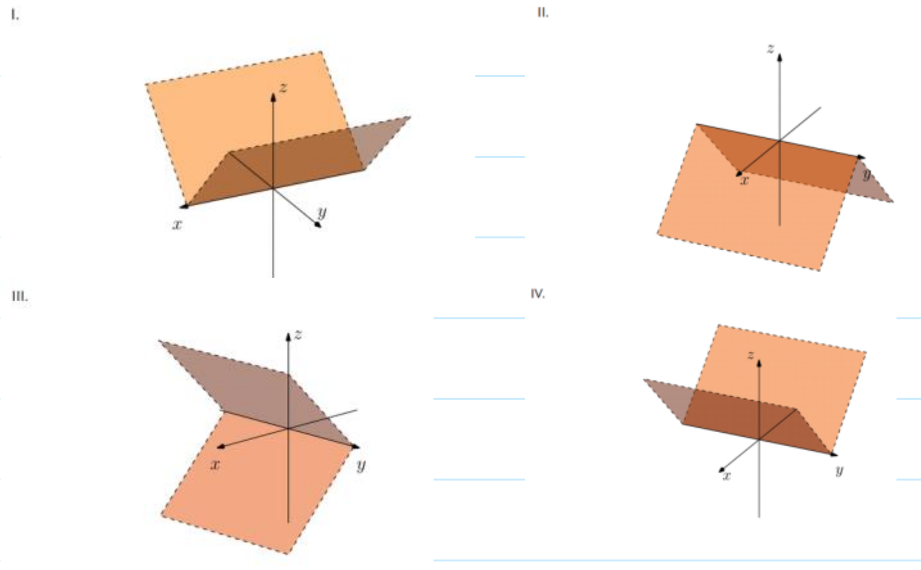
One of the graphs below correctly displays $z = f(x, y) = |x|$ as seen by someone staring down the positive y-axis. Which one is it?



和 y 無關, $|x| = z \Rightarrow \{(x,z) \in \mathbb{R}^2 \mid z = |x|\}$

$\Rightarrow$ 第III張

The graph $z = |x|$ projected onto the xz-plane down the positive y-axis looks like a **V**. Notice how z doesn't depend on y at all. Use this to figure out exactly which of the options below corresponds to the full 3D graph of $z = |x|$.



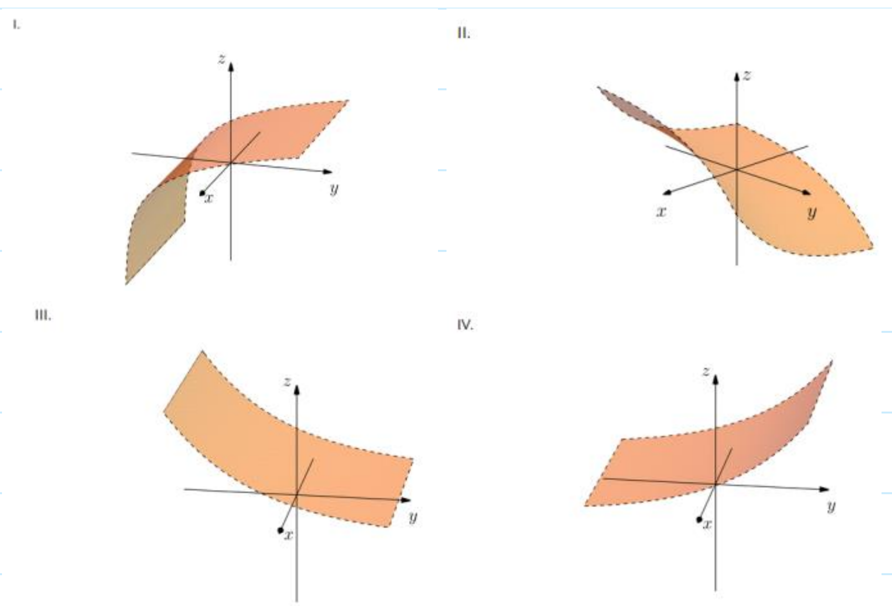
$z = |x|$

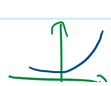
$\Rightarrow (x, y, |x|) \Rightarrow$ IV

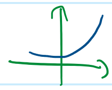
The same idea applies if z doesn't depend on x. Let's get direct experience with this idea by plotting

$$z = e^y.$$

Select the correct 3D graph from the options below.

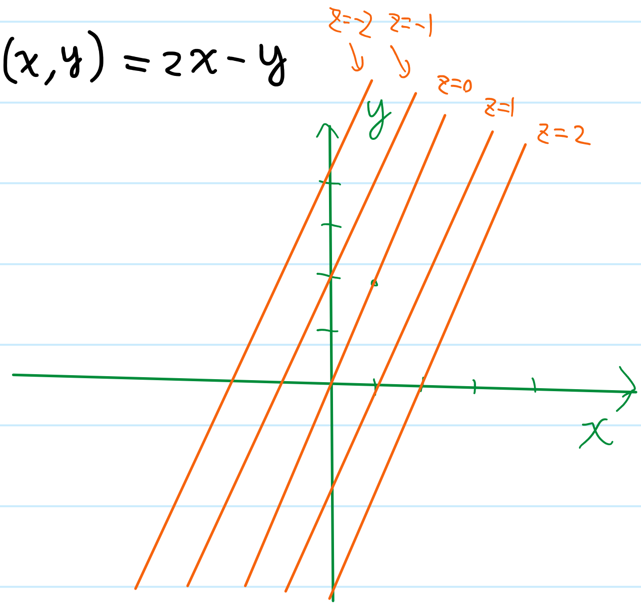


if $y = e^x \Rightarrow$ 

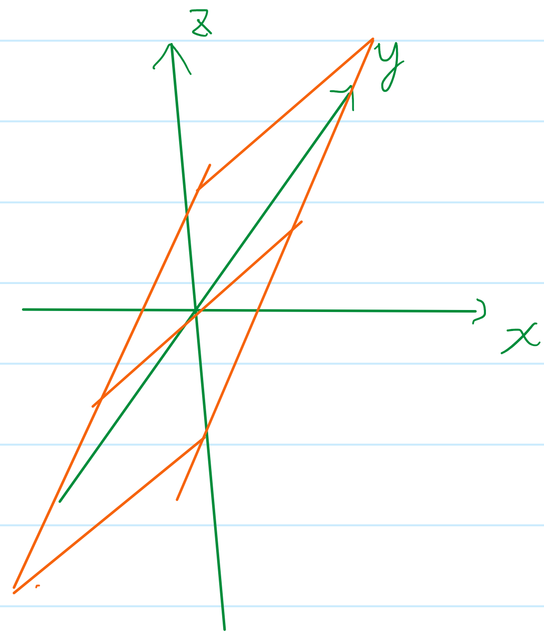
if $y=e^x \Rightarrow$ 

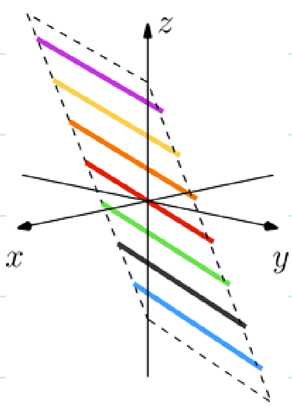
then $z=e^y \Rightarrow$ IV

$f(x,y)=2x-y$



$\Rightarrow$



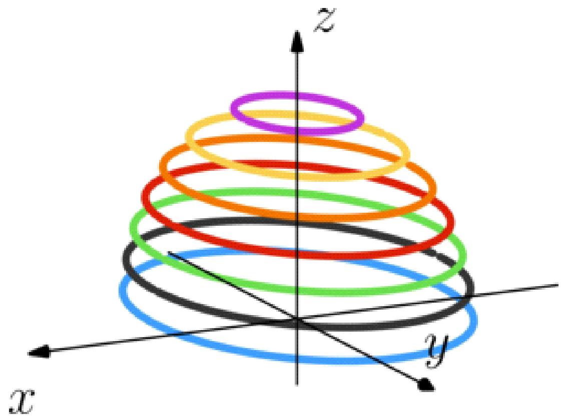


$z=\sqrt{9-2x^2-y^2}$

$\Rightarrow \quad c=\sqrt{9-2x^2-y^2}$

$\Rightarrow \quad c^2=9-2x^2-y^2$

$\Rightarrow \quad 2x^2+y^2=9-c^2$ ← 橢圓

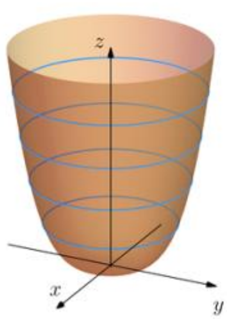


Use horizontal slices to determine the graph of

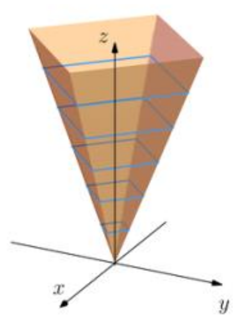
$$f(x,y) = |xy|.$$

Select the best option from below.

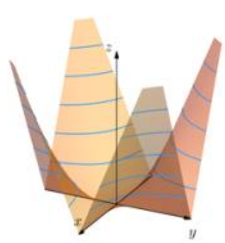
I.



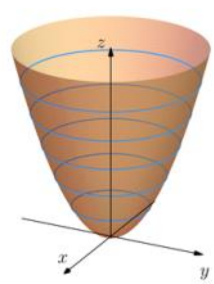
II.

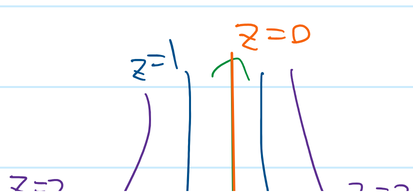


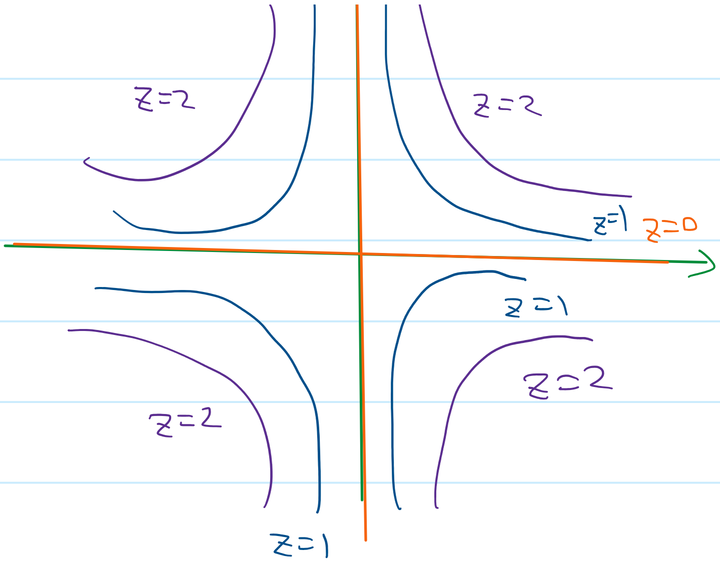
III.



IV.



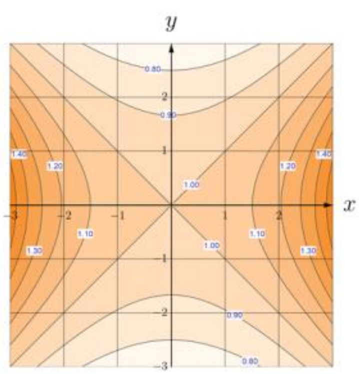




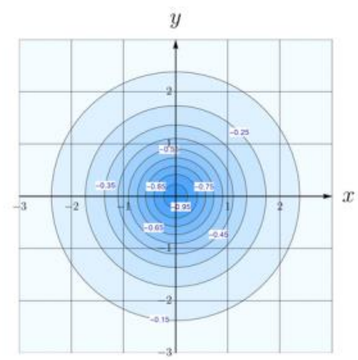
$\Rightarrow$ III

$$T(x,y) = \frac{1}{1+x^2+y^2}$$

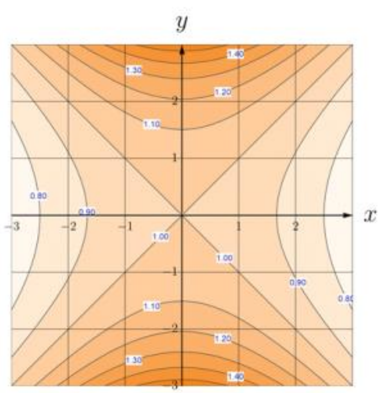
I.



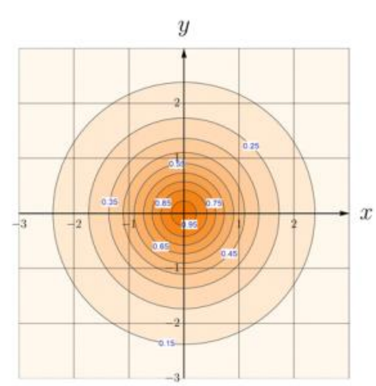
II.



III.



IV.



$c = \frac{1}{1+x^2+y^2}$ $\quad x^2+y^2+1 = \frac{1}{c}$ $\quad x^2+y^2 = \frac{1}{c} - 1 = \left(\sqrt{\frac{1}{c}-1}\right)^2$

for $c<1$ $\quad y$

$1+x^2+y^2$

$\sqrt{\frac{1}{c}-1}$: 圓半徑

for $c<1$,